OVERVIEW OF CURVE FITTING METHODS

General Training

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Agenda

- Overview of the curve-fitting process
- Discuss available curve-fitting methods
- Strengths and weaknesses of each
- Different techniques for understanding test data

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Parameter Estimation

- Objective: construct mathematical model of vibrational properties (parameters) and physical deformation (test article mode shapes)
- Parameter table contains mode shape scaling information
  - Frequency, Damping
  - Residue (amplitude, phase for scaling)
  - Reference, Response locations used for scaling
- Direct correspondence exists between parameter estimation method and mode shape extraction method

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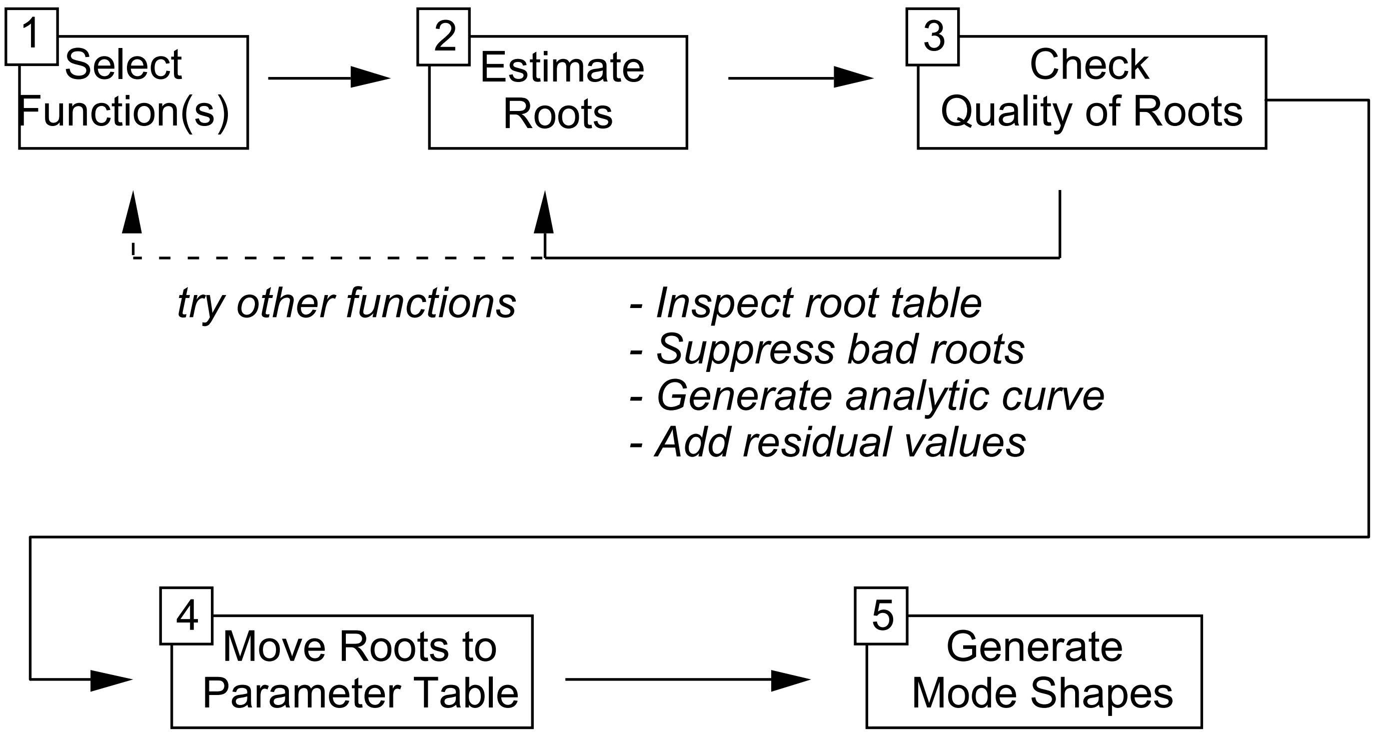
Curve-fitting process



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Two curve-fitting categories

- Single degree of freedom
  - Move response (total response)
  - SDOF Polynomial
  - Circle fit
- Multiple degree of freedom (matrix)
  - Complex exponential
  - Polyreference
  - Direct parameter
  - Orthogonal polyreference

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SDOF is simpler

- Easy techniques to learn and apply
- Results obtained rapidly
- Operates on one mode at a time
- Uncoupled modes are more easily extracted than coupled modes
- Requires sufficient mode separation
- Effective for trouble shooting
- Generally not as accurate as MDOF techniques

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Move (Total) Response

- Not a true curve-fit method
- Single spectral line extraction
- Extract real, imaginary, both, or amplitude
  - Real or complex mode extraction
- This method has benefits
  - Fast method, little user interaction
- This method has drawbacks
  - Need good mode separation and light damping
  - Does not provide a damping estimate

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Move Response Usage

- First build a list of frequencies (parameter table)
  - pick frequencies, search peaks, search valleys
- Create move response shapes
  - Extracts quantity at closest spectral line for each FRF
  - Generates one mode shape for each entry in parameter table
  - Functions used are keyed by first active reference

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SDOF Polynomial

- Fit one mode at a time
- Use one FRF for residue calculation
- This method has benefits
  - Works directly in frequency domain
  - Little user interaction, quick feedback on fit quality
- This method has drawbacks
  - Need good mode separation
  - Not effective for repeated roots
  - Residual contributions from other modes not taken into account

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SDOF Polynomial Usage

- Select a frequency (parameter table)
  - pick frequencies, search peaks, search valleys
- Calculate residue
  - Select coordinate
  - Choose number of spectral lines
  - Repeat coordinate selection until satisfied
- Create SDOF polynomial shape
  - Performs curve-fit on each FRF selected

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Circle Fit

- Fit one mode at a time
- This method has benefits
  - Works directly in frequency domain
  - Best method when frequency shifts occur
  - Can handle closely spaced modes
- This method has drawbacks
  - FRF quality very important; requires good resolution
  - Not effective for repeated roots
  - Can require significant user interaction

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Circle Fit Usage

- Select a frequency (parameter table)
  - pick frequencies, search peaks, search valleys
- Calculate residue
  - Select coordinate
  - Select frequency range
  - Select circle fit options
  - Repeat until satisfied
- Create Circle fit shape
  - Performs curve-fit on each FRF selected

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MDOF is more complicated

- Handle multiple modes at one time
- Use single or multiple FRF
- Can use a single or multiple references
- Can account for residual affects of other modes

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Complex Exponential

- Method utilizes iterative, non least-squares fit in the time domain on a single FRF
- Simplest case of polyreference method
- This method has benefits
  - Fit multiple modes at a time
  - Fits individual FRF well
- This method has drawbacks
  - Not a good method for complicated structures
  - Uses a single function only
  - Sensitive to noise on measurements

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Complex Exponential Usage

- Select a list of frequencies (parameter table)
  - pick frequencies, search peaks, search valleys
- Calculate residue
  - Select coordinate
  - Select FRF
  - Select frequency range and number of poles (2x modes)
  - Repeat coordinate selection until satisfied
- Create complex exponential shape
  - Select number of poles and FRF to fit

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Direct Parameter overview

- Frequency domain approach
- Uses single reference data
- Operates on multiple FRF for a global least squares estimate
- Can fit real or complex modes
- Special case is SDOF polynomial

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Direct Parameter benefits

- Method works in the frequency domain
  - Works better for modes with high damping
- “Correct” number of roots usually obvious
- Accounts for effects of residual terms
- Provides a global estimate of system characteristics
- Well suited to narrow band curve fitting

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Direct Parameter drawbacks

- In general, works over a limited frequency bandwidth
- May require several iterations to extract good parameters
- Sensitive to frequency range selected
  - Use phase angle as a guide

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Direct Parameter Usage

- Build a matrix
  - select multiple FRF (single reference), frequency range
  - Select number of roots
- Select parameters
  - Number of poles
- Calculate residue
  - Loop over multiple FRF
- Create direct parameter shape
  - Select FRF to fit

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Polyreference overview

- Uses single or multiple references and FRF
- Works in time or frequency domain
- Computes “global” set of poles from which to base residues
- Extremely good for high modal density
- Can compute real or complex modes
- Requires some experience and interaction

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Polyreference benefits

- Excellent method for large number of modes
- Works well with closely spaced modes
- Results closely follow MMIF trends
- Global mode estimate
- Frequency and time domain methods available
- Scale modes according to selected reference
- Stability diagram allows easy pole selection

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Polyreference drawbacks

- Requires user judgment and interaction
- Time domain methods not good for high damping
- Must "weed out" computational roots

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Polyreference Usage

- Build a matrix
  - select multiple FRF, frequency range
  - Select number of roots (3-4x number of apparent modes)
- Select parameters
  - Stability plot or number of poles (error chart)
- Calculate residue
  - Loop over multiple FRF
- Create polyreference shape
  - Select FRF to fit

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Orthogonal polyreference

- This method has benefits
  - Frequency domain technique
  - Computational modes generally outside frequency range of interest
  - Multiple reference/response global estimate
  - Allows for hysteretic damping effects
- This method has drawbacks
  - Somewhat limited in frequency range
  - User interaction required

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Orthogonal Polyreference Usage

- Build a matrix
  - select multiple FRF, frequency range
  - Select number of roots
- Select parameters
  - Stability plot or number of poles (error chart)
- Calculate residue
  - Loop over multiple FRF
- Create polyreference shape
  - Select FRF to fit

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Choose the right method

- Depends on type of data
  - Frequency shifts evident?
  - Is data lightly or highly damped?
  - Do repeated roots exist?
- Generally, start with polyreference
- Use other techniques to supplement polyreference

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MIF show frequencies

- Used to identify resonant frequencies and how well modes are excited
- Apparent modes indicated by MIF minima
- Multivariate MIF are used for multiple references
  - Shows evidence of repeated roots
  - Generates force vectors to “tune” normal modes
- May indicate modes not readily identifiable from FRF
- Useful to identify frequencies for parameter estimation

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Summary

- Several curve fitting techniques available
- Best method depends on test data
- Use Mode Indicator Functions to identify mode frequencies

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