Overview of Modal Theory

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General Training

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Why Modal Analysis?

- Characterize the fundamental dynamic characteristics of a structure.
  - Natural frequencies
  - Damping
  - Mode Shapes
- Understand and predict the response of a structure to an external excitation.
  - Damage detection
  - Design of required vibration isolation systems

What are Modal Models?

- Modal models represent a structure with
  - Modal frequencies
  - Modal damping values
  - Mode Shapes

  } ⇒ modal parameters

- Modal models do not have a mass, stiffness, or damping matrix as does a FEM.
- Types of modal models:
  - Analytical: Complex or normal modes analysis.
  - Experimental: Perform a modal survey.

Analytical Modal Analysis

- Discretize a structure with a mathematical model.
  - Lumped mass/spring models
  - Finite element models (FEM)
- N DOF system with viscous damping yields N coupled 2nd order ordinary differential equations.
- Equation of motion is given by:

$$[M][\ddot{q}] + [C][\dot{q}] + [K][q] = [f]$$

Analytical Equation of Motion

- [M] = mass matrix
- [C] = viscous damping matrix
- [K] = stiffness matrix
- [q] = displacement vector
- [f] = force vector
- If C is proportional to M and K then real normal modes.
- If C is not proportional to M and K then complex modes.

$$[M][\ddot{q}] + [C][\dot{q}] + [K][q] = [f]$$

Alternate Form of Equation of Motion

- **Multiplying through by the inverse of the mass matrix yields:**

$$[\ddot{q}]+[M]^{-1}[C][\dot{q}]+[M]^{-1}[K][q]=[M]^{-1}[f]$$

- **Further rearranging yields**

$$\underbrace{\begin{bmatrix}[\dot{q}]\\ [\ddot{q}]\end{bmatrix}}_{\dot{z}}=\underbrace{\begin{bmatrix}0_{NxN} & I_{NxN}\\ -[M]^{-1}[K] & -[M]^{-1}[C]\end{bmatrix}}_{A}\underbrace{\begin{bmatrix}[q]\\ [\dot{q}]\end{bmatrix}}_{z}+\underbrace{\begin{bmatrix}0_{Nx1}\\ [M]^{-1}\end{bmatrix}}_{B}[f]$$

Complex Eigenvalue Analysis

- Let $z = Z e^{\lambda t}$ and set $f = 0$ then

  $\lambda Z e^{\lambda t} = A Z e^{\lambda t}$ and thus $\left[\lambda I - A\right] Z = 0$.

- Note that $\left[\lambda I - A\right] Z = 0$ only has a solution if

  $\left|\lambda I - A\right| = 0$ which yields the characteristic equation.

  λ = eigenvalue (modal frequency & damping),

  Z = eigenvector (complex mode shape)

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Real Eigenvalue Analysis

- **Let** $q = \phi e^{\lambda t}$
- **If C is such that** $[\phi]^T [C][\phi]$ **is diagonal (C = aM +bK) then the eigenvalue problem simplifies to**

$$\lambda^2 M \phi e^{\lambda t} + K \phi e^{\lambda t} = 0$$

$$\left[\lambda^2 M + K\right] \phi = 0$$

λ **= eigenvalue (modal frequency & damping)**

$$\left|\lambda^2 M + K\right| = 0$$

Real normal modes

ϕ **= eigenvector (real mode shape)**

Real Normal Modes

- Mode shapes are orthogonal to M and K
  - $[\phi]^T M [\phi], [\phi]^T K [\phi]$ are diagonal matrices

- Mode shapes can be scaled so they are orthonormal to M

$$[\phi]^T M [\phi] = I_{NxN}, \quad [\phi]^T K [\phi] = \begin{bmatrix} \ddots & & \\ & \omega_n^2 & \\ & & \ddots \end{bmatrix}$$

Modal Coordinates

- Let $q = \phi \xi$ and let $[C] = \alpha [M] + \beta [K]$

  where ξ is the generalized (modal coordinate).

$$[M][\phi][\ddot{\xi}] + [C][\phi][\dot{\xi}] + [K][\phi][\xi] = [f]$$

- Premultiplying by $[\phi]^T$ yields

$$[\phi]^T [M][\phi][\ddot{\xi}] + [\phi]^T [C][\phi][\dot{\xi}] + \cdots$$

$$+ [\phi]^T [K][\phi][\xi] = [\phi]^T [f]$$

Modal Equation of Motion

$$[\phi]^T[M][\phi][\ddot{\xi}] + [\phi]^T[C][\phi][\dot{\xi}] + \cdots$$
$$+ [\phi]^T[K][\phi][\xi] = [\phi]^T[f]$$

$$[\ddot{\xi}] + \begin{bmatrix} \ddots & & \\ & 2\zeta\omega_n & \\ & & \ddots \end{bmatrix}[\dot{\xi}] + \begin{bmatrix} \ddots & & \\ & \omega_n^2 & \\ & & \ddots \end{bmatrix}[\xi] = [\phi]^T[f]$$

- **N 2nd order uncoupled modes!!!**

Modal Damping Matrix

- Often times damping matrix is formed in modal coordinates.
  - Assign individual damping values to each mode.
  - Assign a “conservative” global damping value to a group of modes.
- Modal damping values typically come from test data.

Experimental Modal Analysis

- Experimental modal analysis extracts (identifies) modal parameters from test data.
- Test data types
  - Raw time histories
  - FRF estimated from raw time histories
- Assume structure is linear and time-invariant (LTI)
  - “Mild” nonlinearites can also be characterized

Extraction of Modal Parameters

- Extraction ("Identification") techniques
  - Time domain techniques (operate directly on the raw time histories)
  - Time domain techniques (operate on the impulse response function derived from the estimated FRF)
  - Frequency domain techniques (operate on FRF estimated from the raw time histories)
  - SDOF techniques (single FRF)
  - MDOF techniques (multiple FRF, global estimator)

Why an Experimental Modal Model?

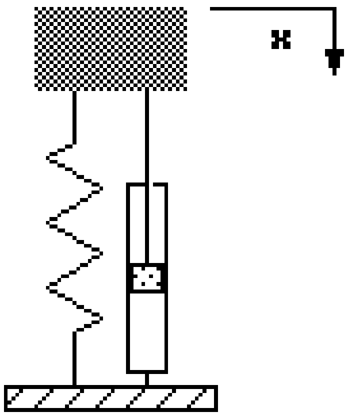
- An accurate FEM may not exist.
- Damping is difficult to include accurately in a FEM.
- Joints with free-play are very difficult to model.
  - Effective joint stiffness?
  - Effective joint damping?
- Health monitoring and damage detection
  - Significant shifts in modal parameters indicate stiffness and or mass changes in the structure.
  - However, very localized damage may not be easily detected.

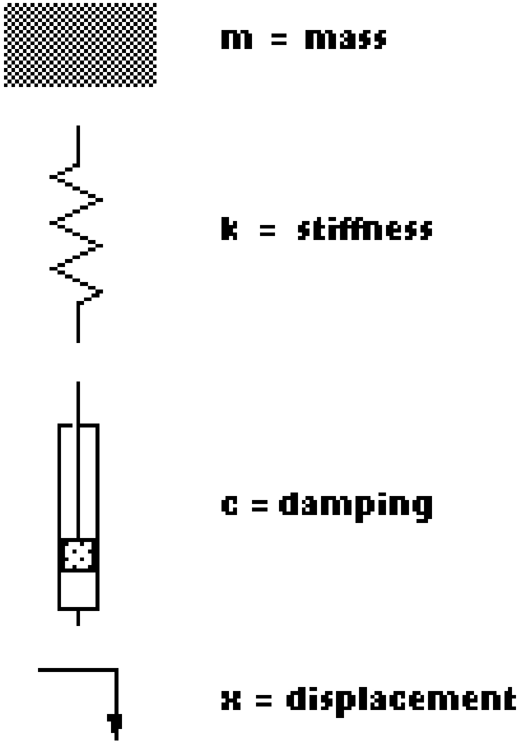
FEM Correlation

- Experimental modal model can be used to update the analytical model (FEM).
  - Correlate test and analytical mode shapes.
  - Correlate test and analytical FRF.
    - Modify mass and stiffness properties of individual elements in the FEM.
- Correlated FEM can be used for
  - Flutter analysis
  - Design of feedback control systems.
    - Robust stability and performance

Single Degree of Freedom Oscillator

- Single degree of freedom (SDOF) oscillator





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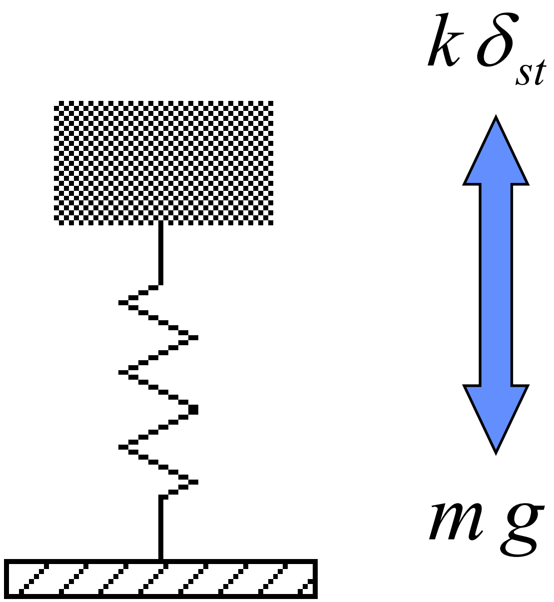
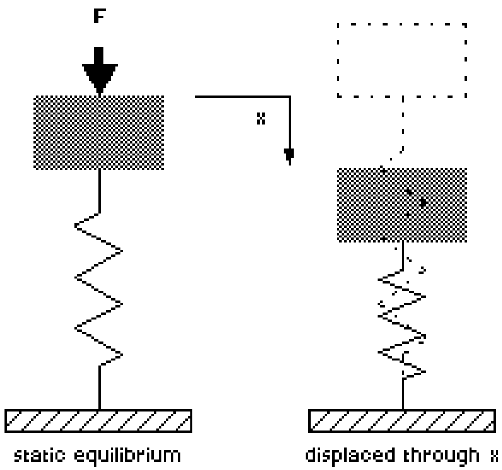
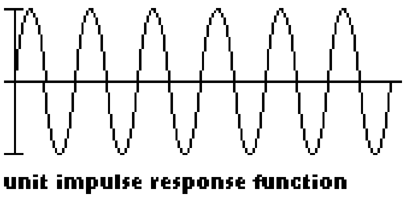
SDOF Under Static Loading

- The weight of the mass (mg) is equal to the force created in the spring ($k\ \delta_{st}$)
  - Static deflection δ_{st}
  - Spring stiffness, k
- Mass in equilibrium



Free Vibration





$$F_k = k\left(\zeta_{st} + x\right)$$

$$F = m\,g - F_k = -k\,x$$

- F_k - Force in the spring
- F - Total force acting on the mass

Setting Up Equations Of Motion

- Newton's 2nd Law - F = ma

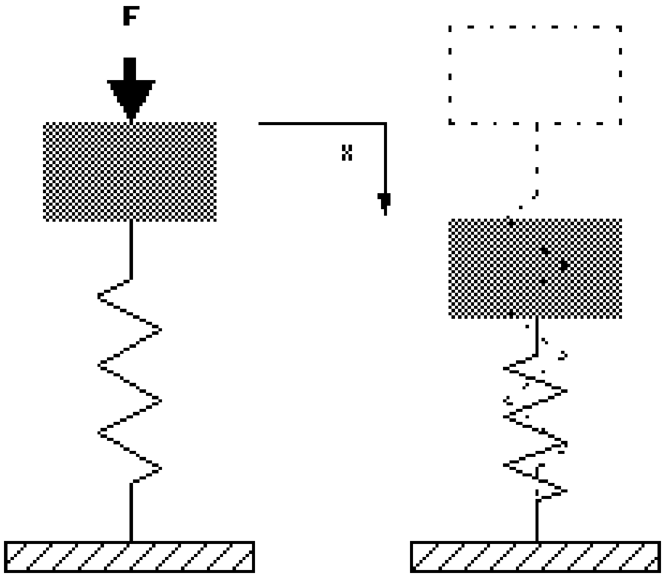
$$m\,\ddot{x} = -k\,x$$

$$m\,\ddot{x} + k\,x = 0$$

- Force term is zero for free vibration
  - mass is simply displaced by a unit impulse
  - no other forces applied to the structure
- Note that while the terms for mass and stiffness are present, there is no term for damping since an undamped structure has a damping factor of zero

Undamped Forced Vibration

- **Excitation force, F, applied**
  - F sin(ω t)



$$F \sin \omega t + m g - k\left(\delta_{st} + x\right) = m \ddot{x}$$

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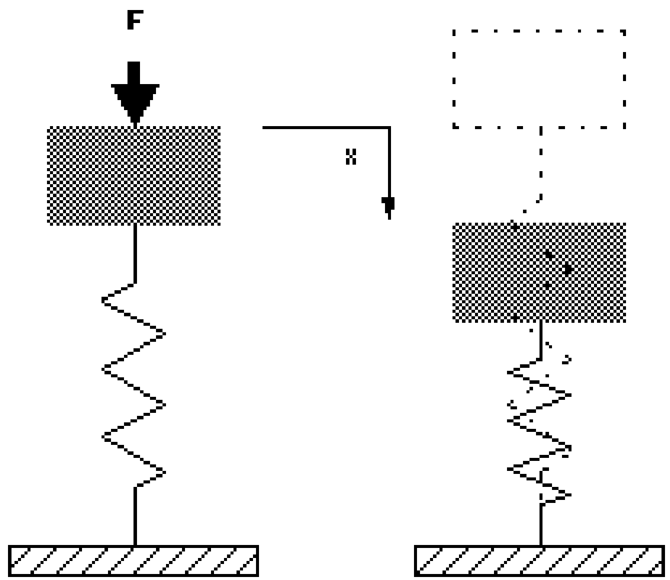
Resulting Equation Of Motion

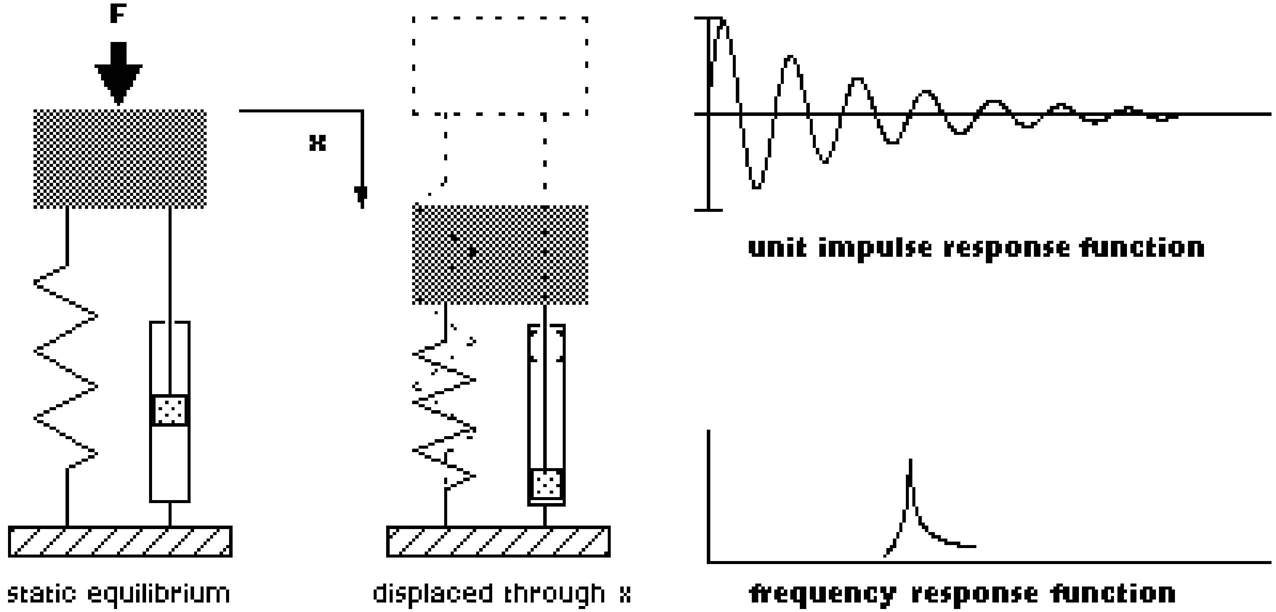
- Excitation force, F, applied
  - F sin(ωt)
  - $mg = k\delta_{st}$



$$m\ddot{x} + kx = F \sin \omega t$$

Damped Free Vibration

- Excitation force, F, applied
  - F $\sin(\omega t)$
  - $mg = k\delta_{st}$



Resonance Frequency

- Resonance frequency is the frequency at which stiffness and inertial forces of the structure cancel each other out as the amplitude peaks.
- Peaks in a frequency response function are used to identify the modes of a structure.
- Damping is proportional to the velocity of the mass.
- Equation of motion for damped free vibration is given by:

$$m g - k\left(\delta_{st} + x\right) - c\dot{x} = m\ddot{x}$$

$$m\ddot{x} + c\dot{x} + kx = 0$$

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Damped Forced Vibration

- **Damped forced vibration equation of motion is given by:**

$$m\ddot{x} + c\dot{x} + kx = F\sin\omega t$$

$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = \frac{F}{m}\sin\omega t$$

$$\ddot{x} + 2\varsigma\omega_n\dot{x} + \omega_n^2 x = \frac{F}{m}\sin\omega t$$

SDOF Modal Parameters

$$\ddot{x} + 2\varsigma\omega_n\,\dot{x} + \omega_n^2\,x = \frac{F}{m}\sin\omega t$$

$$\varsigma = \frac{c}{2\omega_n m} = \frac{c}{2\sqrt{k\,m}} = \text{damping coefficient}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \text{undamped natural frequency}$$

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Characteristic Equation & Poles

$$\ddot{x} + 2\varsigma\,\omega_n\,\dot{x} + \omega_n^2\,x = \frac{F}{m}\sin\omega\,t$$

- Set f = 0 and substitute $x = X\,e^{\lambda t}$ yields

$$\left[\lambda^2 + 2\varsigma\,\omega_n\,\lambda + \omega_n^2\right]X = 0$$

- The poles are the roots of the characteristic equation:

$$\lambda^2 + 2\varsigma\,\omega_n\,\lambda + \omega_n^2 = 0$$

Undamped SDOF Oscillator

- If $\varsigma = 0$ then the poles (roots of the characteristic equation) are pure imaginary complex conjugate pair:

$$\lambda_{1,2} = \pm j\omega_n$$

- The transient (homogenous) solution is of the form:

$$x(t) = C_1 e^{j\omega_n t} + C_2 e^{-j\omega_n t}$$

sine wave

Under Damped SDOF Oscillator

- If $0 < \varsigma < 1$ then the poles (roots of the characteristic equation) are a complex conjugate pair:

$$\lambda_{1,2} = -\zeta\,\omega_n \pm j\,\omega_n\sqrt{1-\zeta^2}$$

- Damped natural frequency $\omega_d = \omega_n\sqrt{1-\zeta^2}$
- The transient (homogenous) solution is of the form:

$$x(t) = C_1\,e^{\left(-\zeta\,\omega_n + j\,\omega_n\sqrt{1-\zeta^2}\right)t} + C_2\,e^{\left(-\zeta\,\omega_n - j\,\omega_n\sqrt{1-\zeta^2}\right)t}$$

exponentially decaying sinusoid

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Critically Damped SDOF Oscillator

- If $\varsigma = 1$ then the poles (roots of the characteristic equation) are a real and repeated:

$$\lambda_{1,2} = -\omega_n$$

- The transient (homogenous) solution is of the form:

$$x(t) = C_1 e^{\omega_n t} + C_2 t e^{\omega_n t}$$

Over Damped SDOF Oscillator

- If $\varsigma > 1$ then the poles (roots of the characteristic equation) are a real and distinct:

$$\lambda_{1,2} = -\zeta \omega_n \pm \omega_n \sqrt{\zeta^2 - 1}$$

- The transient (homogenous) solution is of the form:

$$x(t) = C_1 e^{\left(-\zeta \omega_n + \omega_n \sqrt{\zeta^2 - 1}\right)t} + C_2 e^{\left(-\zeta \omega_n - \omega_n \sqrt{\zeta^2 - 1}\right)t}$$

SDOF FRF

- **The force to displacement FRF is given by:**

$$\frac{X(j\omega)}{F(j\omega)} = \frac{1/m}{(-\omega^2 + j2\zeta\omega_n\omega + \omega_n^2)}$$

- **The force to acceleration FRF is given by:**

$$\frac{A(j\omega)}{F(j\omega)} = \frac{-\omega^2/m}{(-\omega^2 + j2\zeta\omega_n\omega + \omega_n^2)}$$

- **Recall:** $\frac{V(j\omega)}{F(j\omega)} = j\omega\frac{X(j\omega)}{F(j\omega)}, \frac{A(j\omega)}{F(j\omega)} = j\omega\frac{V(j\omega)}{F(j\omega)}$

SDOF FRF Characteristics

- At frequencies below the undamped natural frequency

$$\left.\frac{X(j\omega)}{F(j\omega)}\right|_{\omega<<\omega_n} \approx \frac{1}{k}$$

"stiffness line"

- At the undamped natural frequency

$$\angle \frac{X(j\omega_n)}{F(j\omega_n)} = -90\,\mathrm{deg}$$

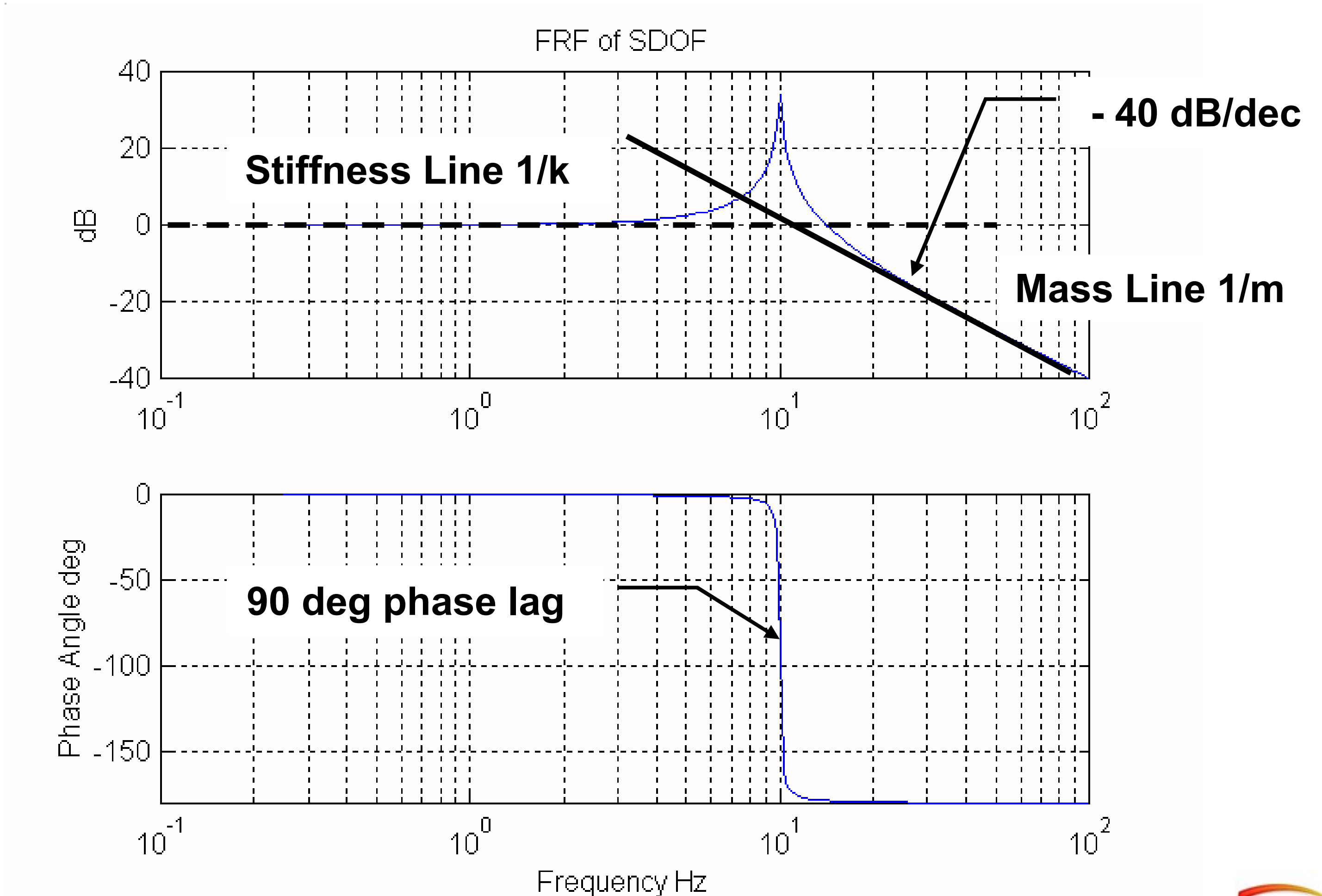
FRF is pure imaginary

- At frequencies above the undamped natural frequency

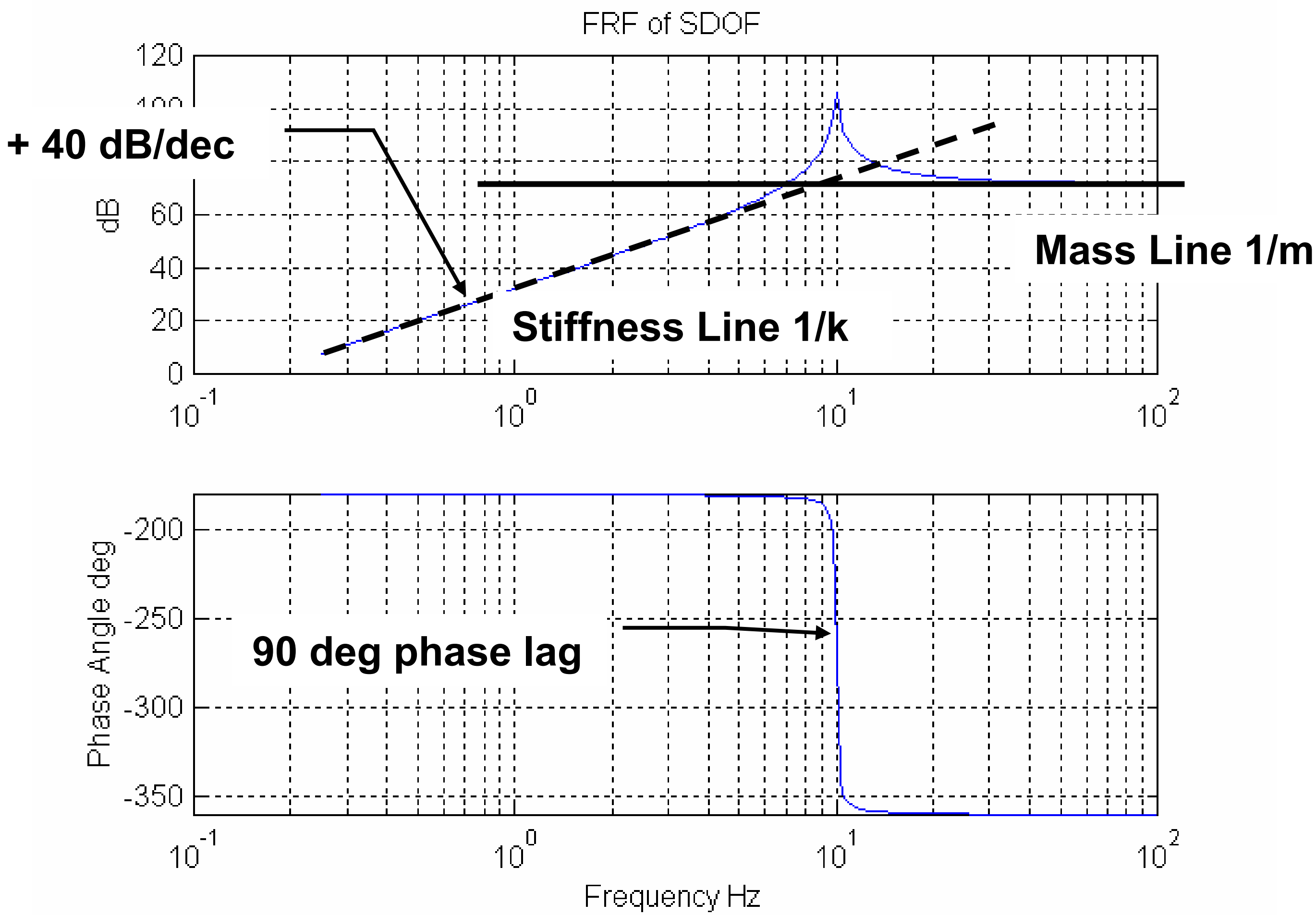
$$\left.\frac{X(j\omega)}{F(j\omega)}\right|_{\omega>>\omega_n} \approx \frac{1}{m\omega^2}$$

"mass line"

SDOF Displacement FRF



SDOF Acceleration FRF



SDOF FRF Characteristics

- **For underdamped SDOF oscillators**

$$\omega_{\max} = \omega_n \sqrt{1-2\zeta^2} \qquad \frac{X(j\omega_{\max})}{F(j\omega_{\max})} = \frac{1/m}{2\zeta\omega_n^2\sqrt{1-2\zeta^2}}$$

- **If damping is very small then the frequency of the resonance peak is close to the undamped natural frequency**

$$\omega_{\max} \approx \omega_n \qquad \frac{X(j\omega_{\max})}{F(j\omega_{\max})} \approx \frac{1}{2\zeta\, m\, \omega_n^2}$$

SDOF Residues

- Partial fraction expansion of the force to displacement FRF is given by:

$$\frac{X(j\omega)}{F(j\omega)} = \frac{1/m}{(-\omega^2 + j2\zeta\omega_n\omega + \omega_n^2)}$$

$$= \frac{A_1}{(j\omega - \lambda_1)} + \frac{A_2}{(j\omega - \lambda_2)}$$

- A_1, A_2 are the residues.

Underdamped SDOF Residues

- **Partial fraction expansion of the FRF can be written as**

$$\frac{X(j\omega)}{F(j\omega)} = \frac{1/m}{(-\omega^2 + j2\zeta\omega_n\omega + \omega_n^2)}$$

$$= \frac{A_1}{(j\omega + \zeta\omega_n - j\omega_d)} + \frac{A_2}{(j\omega + \zeta\omega_n + j\omega_d)}$$

$$= \frac{A_1}{(j\omega + \zeta\omega_n - j\omega_d)} + \frac{A_1^*}{(j\omega + \zeta\omega_n + j\omega_d)}$$

- **The residues are** $A_1 = \frac{-j}{2m\omega_d}$ $\quad A_2 = \frac{+j}{2m\omega_d}$

SDOF Impulse Response Function

- The (displacement) impulse response is given by:

$$g(t) = A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t}, t \geq 0$$

- Note that the residues A_1, A_2 are the scaling coefficients.

- For an under damped system the impulse response is:

$$g(t) = \frac{e^{-\zeta \omega_n t}}{m \omega_d} \sin(\omega_d t), t \geq 0$$

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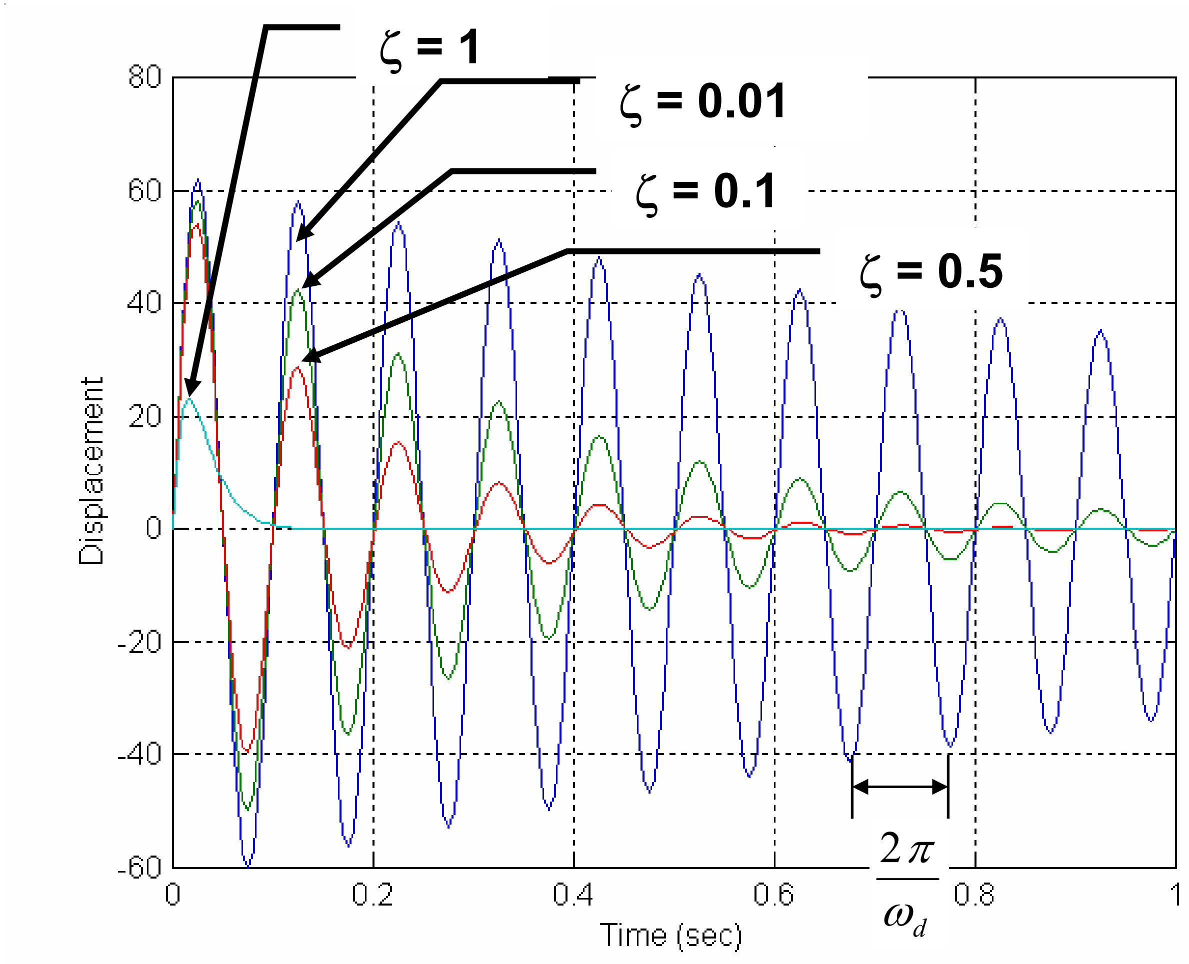
SDOF Impulse Response Function



Convolution Integral

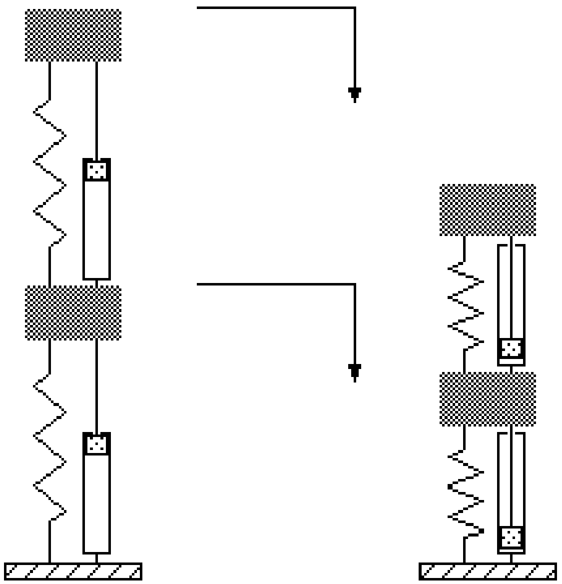
- The response of a system to an arbitrary input is given by:

$$x(t)=\int_{0}^{t} g(t-\tau)\, f(t)\, dt$$

- This is also known as Duhammel's integral.

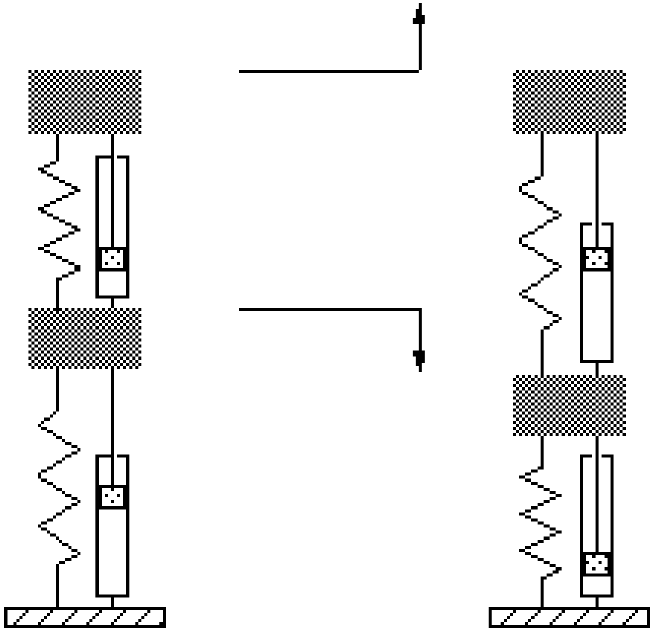
Two DOF System

- A structure with two masses (i.e. 2 DOF) behaves in a more complex way.
- 1st mode has both masses oscillating in the same direction (in-phase)
  - Relatively low frequency and a relatively high amplitude.



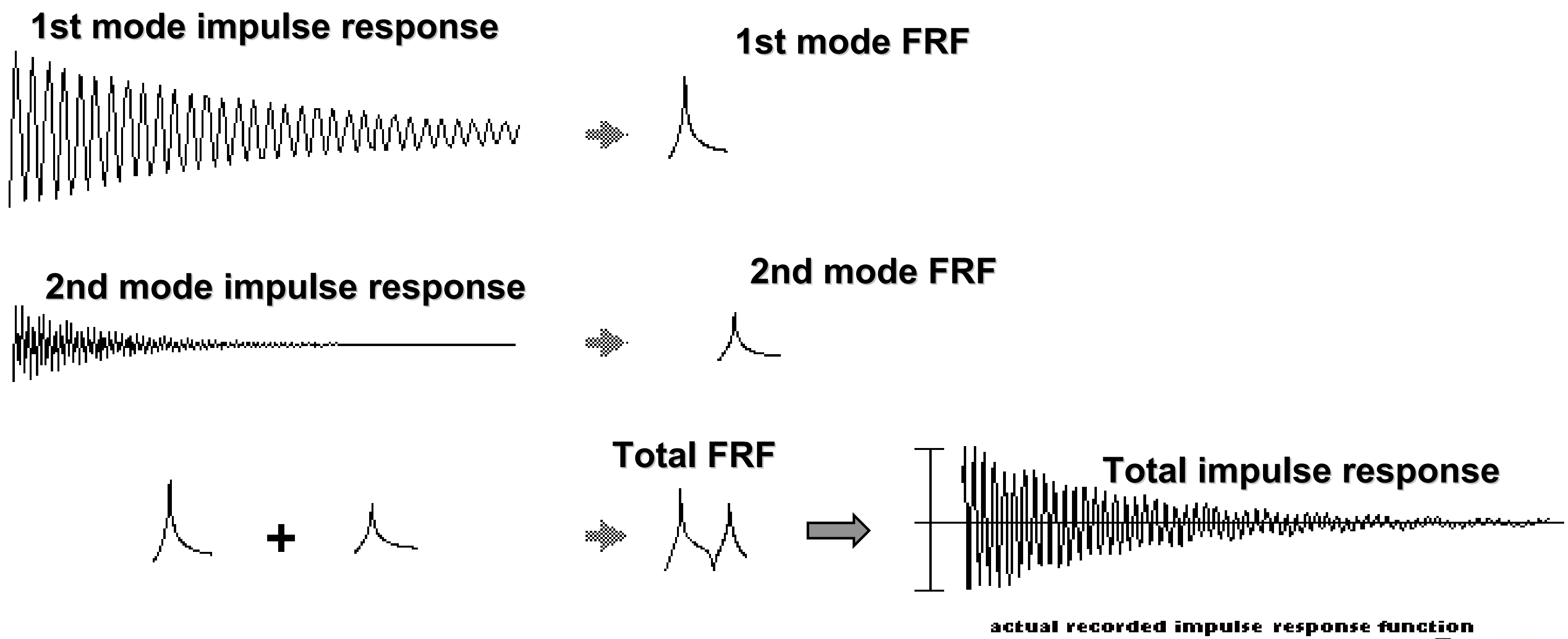
Two DOF System

- 2nd mode has both masses oscillating in opposite directions (180 degrees out-of-phase)
  - Relatively high frequency and a relatively low amplitude.



Two DOF Impulse Response

- The total impulse response is the summation of the impulse responses of both modes.
  - Exponentially decaying sinusoidal oscillations



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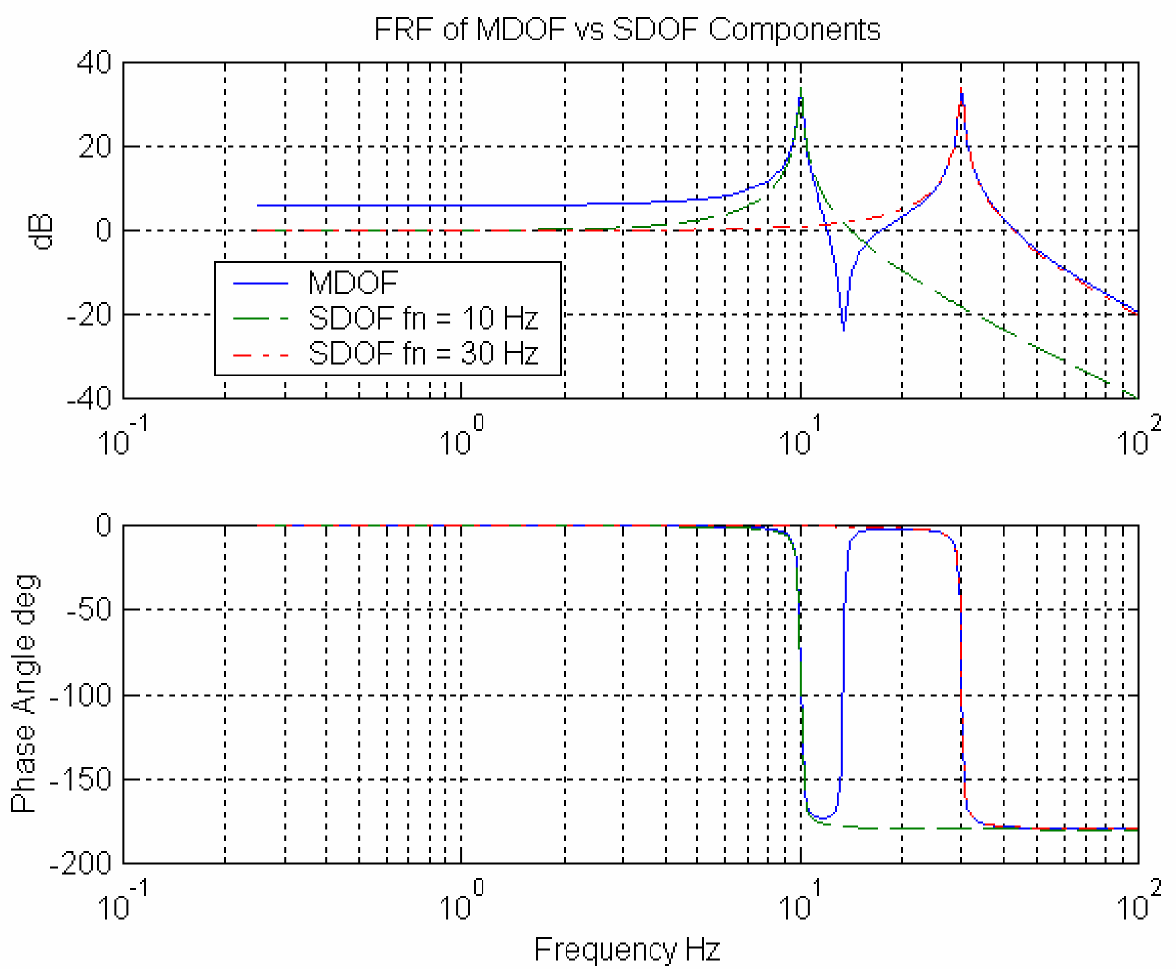
2 SDOF Oscillators in Parallel



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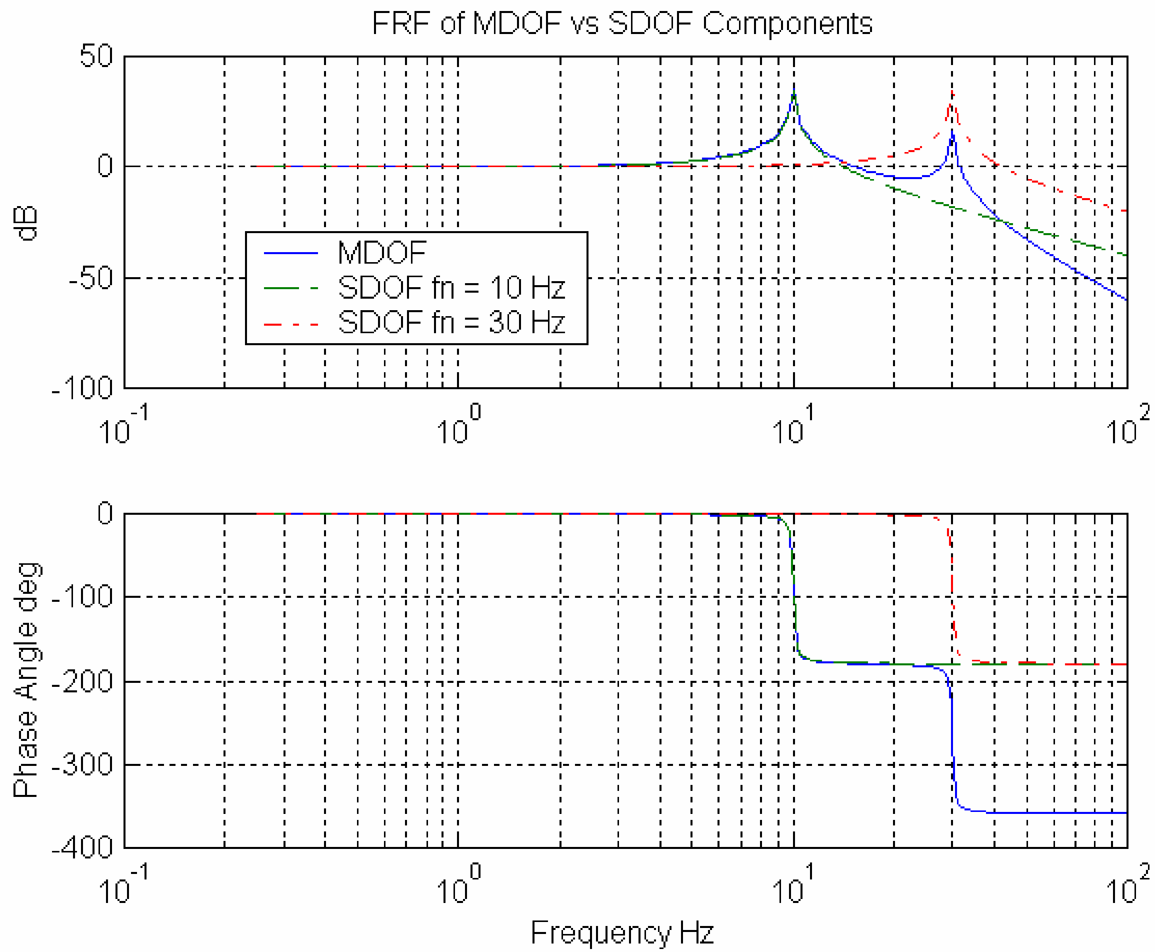
2 SDOF Oscillators in Series



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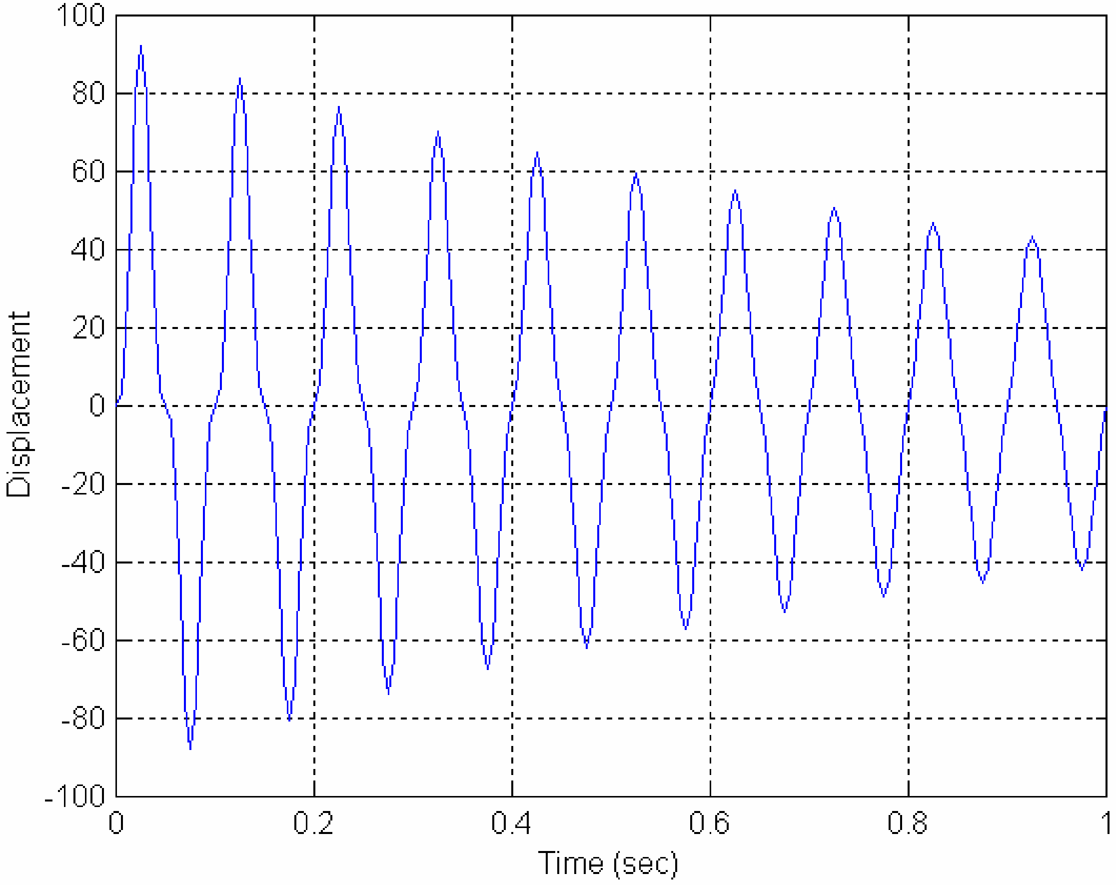
2 SDOF Oscillators in Series



Residues of N DOF Oscillator

- **Partial fraction expansion of the FRF can be written as**

$$\frac{X_i(j\omega)}{F_k(j\omega)} = \sum_{r=1}^{2N} \frac{A_{rik}}{(j\omega - \lambda_r)}$$

- **Where i is the response location and k is the force input location.**
- **If all modes are under damped then**

$$\frac{X_i(j\omega)}{F_k(j\omega)} = \sum_{r=1}^{N} \frac{A_{rik}}{(j\omega + \zeta_r \omega_{n_r} - j\omega_{d_r})} + \frac{A^*_{rik}}{(j\omega + \zeta_r \omega_{n_r} + j\omega_{d_r})}$$

N DOF Impulse Response Function

- The (displacement) impulse response function is given by:

$$g_{ik}(t) = \sum_{r=1}^{2N} A_{rik}\, e^{\lambda_r t}, t \geq 0$$

- Note that the residues are the scaling coefficients.

$$g(t) \quad \underset{\text{Inverse Fourier}}{\overset{\text{Fourier Transform}}{\Rightarrow \atop \Leftarrow}} \quad \frac{X(j\omega)}{F(j\omega)}$$

Modal Parameter Extraction Options

- SDOF techniques (single FRF)
  - SDOF polynomial (least-squares fit of with a rational polynomial)
  - Circle fit (fit a circle of an FRF near a particular resonance peak).
- MDOF
  - Complex exponential
  - Polyreference (time-domain & frequency domain)
  - Direct parameter

Extracting Modal Parameter Check List

- Data quality check
- Preliminary estimate of number of modes
- Preliminary modal extraction
- Checking quality of test mode shapes
  - MAC
  - Ortho
  - Synthesized FRF vs measured FRF
- Refitting “difficult” modes

Data Quality Check

- Very important before starting any modal extraction is to ensure the “quality” of the test data.
- Check list:
  - Did any data channels drop out?
  - Were there any data channels that did not have any significant response?
    - Check the coherence and auto spectrums
  - Do the drive point FRF have the right phase characteristics?

Normal Mode Indicator Function

- Normal mode indicator function (NMIF) uses a single reference.

$$\frac{\sum\left(\left|\mathrm{Real}\left(H\right)\right|\left|H\right|\right)}{\sum\left(\left|H\right|^{2}\right)}$$

  – Valleys show where the real component of the FRF is "minimized" (ie 90 deg phase shift)
  – Valleys indicate real normal modes.
  – Localized and low amplitude modes can be seen.
  – Not well suited to complex modes.

Power Spectrum MIF

- **Power spectrum MIF**

$$\sum \left(H\, H^{*} \right)$$

  - **Peaks indicate modes.**
  - **Localized or lower amplitude modes are often lost.**

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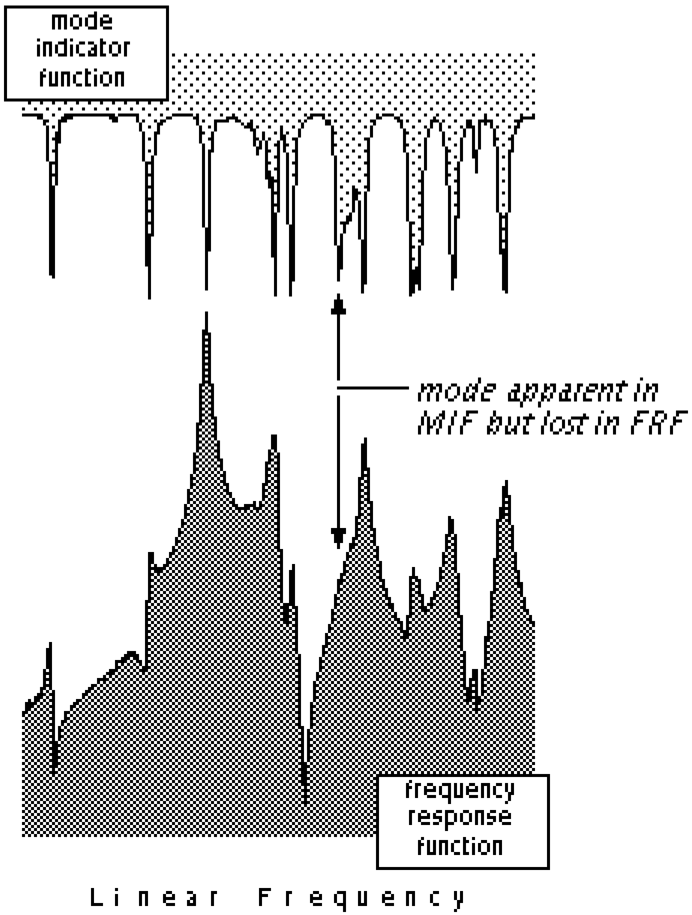
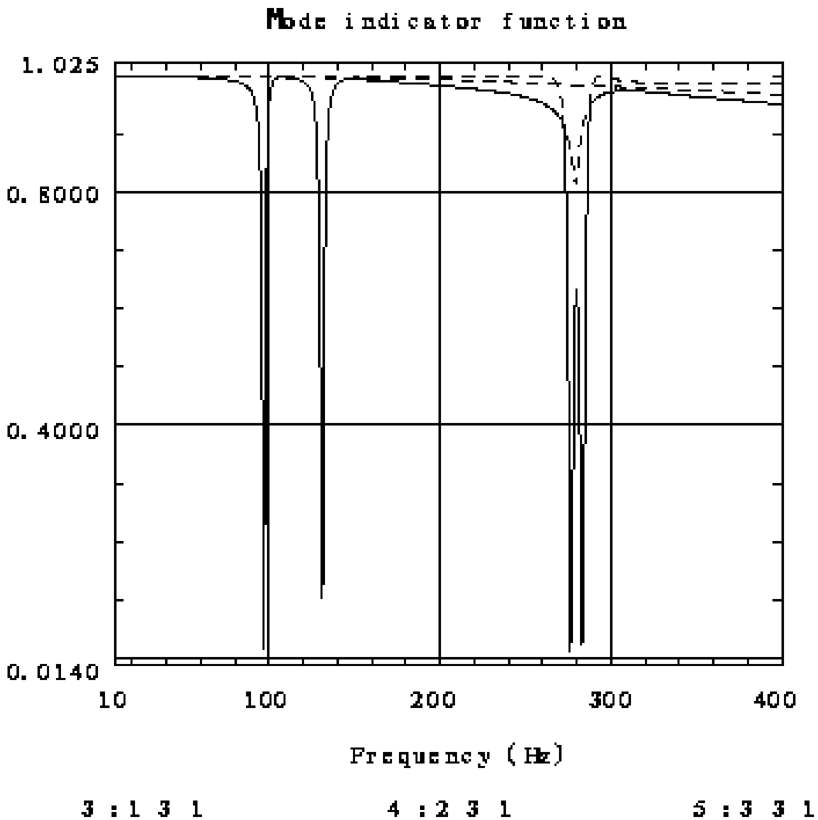
Multivariate Mode Indicator Function

- **Multivariate mode indicator function (MMIF) is similar to NMIF except it can use multiple references.**
  - **Valleys show where the real component of the FRF is "minimized" (ie 90 deg phase shift)**
  - **Valleys indicate real normal modes.**
  - **Localized and low amplitude modes can be seen.**
  - **Has the capability of indicating repeated roots (ie multiple modes at the same frequency).**
  - **Not well suited to complex modes.**

MIF vs FRF



mode indicator function
mode apparent in MIF but lost in FRF
frequency response function
Linear Frequency



Mode indicator function
1.025
0.8000
0.4000
0.0140
10
100
200
300
400
Frequency (Hz)
3 : 1 3 1
4 : 2 3 1
5 : 3 3 1

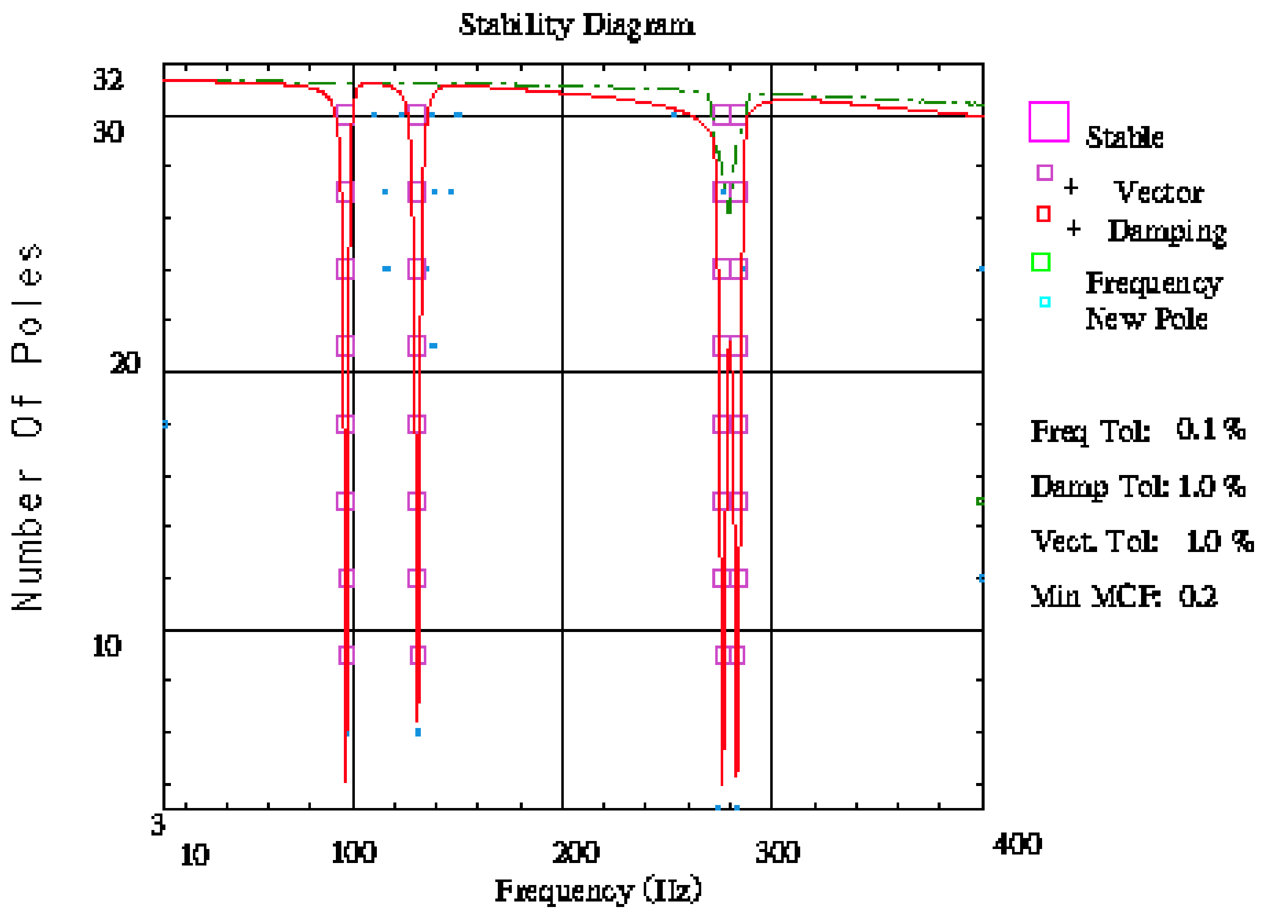
Preliminary Number of Modes

- Compute multivariate mode indicator functions (MMIF) using all “good” references and all “good” responses.
  - Provides a preliminary number and density of modes
  - Shows if there are possibly any repeated modes
- Compute MMIF for each individual reference and overlay with its drive point FRF.
  - Shows whether the drive points excited all of the modes.
- Tag resonance peaks of the FRF.

Preliminary Pole Estimation

- Perform a "first cut" pole estimation using polyreference.
  - Typically choose a correlation matrix size that is 4 times the number of poles you estimate from the MMIF.
  - Overlay the drive point FRF and or the MMIF with the correlation matrix.
  - Are the race track patterns of the pole estimates distinct and consistent with the MMIF and drive point FRF?
  - Check pole estimates for consistent damping values.
- Do not worry about going after every mode.
- Select poles corresponding to the dominant modes.

Overlay Stability Diagram & MMIF



Calculating Residues and Shapes

- Calculate the residues at "key" locations around the structure.
  - Do not need to calculate residues at every location.
- Do the FRF generated when calculating residues "match" the measured FRF?
  - Should be good agreement at the dominant resonance peaks of the selected modes.
  - Need to keep in mind the effect of residual inertance and compliance.
- Be sure to include the drive point FRF when computing shapes if synthesized FRF are desired.

Synthesizing FRF

$$\frac{X_m(j\omega)}{F_n(j\omega)} = \sum_{r=1}^{N} \left(\frac{\phi_{rm}\phi_{rn}}{\phi_{ri}\phi_{rk}} \right) \left(\frac{A_{rik}}{(j\omega + \zeta_r \omega_{n_r} - j\omega_{d_r})} + \frac{A^*_{rik}}{(j\omega + \zeta_r \omega_{n_r} + j\omega_{d_r})} \right)$$

ϕ_{rm} = rth mode, mode shape coefficient of mth response dof

ϕ_{rm} = rth mode, mode shape coefficient of nth reference dof

ϕ_{ri} = rth mode, mode shape coefficient for the response dof in the parameter set

ϕ_{rk} = rth mode, mode shape coefficient for the reference dof in the parameter set

A_{rik} = the amplitude in the parameter set

11995 El Camino Real, Suite 200, San Diego, California 92130 Telephone 877.720.8442 Facsimile 858.792.8932 www.ata-e.com

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Test Mode Shape Verification

- Visually inspect mode shapes.
  - Do the low frequency modes look reasonable?
  - If not may have data channels mixed up.
- Compute MAC (Ortho if analytical mass matrix is available).
  - Are modes independent / orthogonal?
  - Highly coupled modes may be due to
    - Repeated roots (eigen pair)
    - Mode(s) of uninstrumented component(s)
    - Poor modal extraction

Refine Modal Parameter Extraction

- Based upon the calculated residues and test mode shapes may need to go back and identify (re-identify) modes.
- Can use polyreference with a narrow frequency band.
  - Both the time-domain and frequency-domain options use the frequency-domain formulation for pole extraction.
  - Time-domain polyreference is well suited to identifying lightly damped modes.
- If mode(s) are more heavily damped the frequency-domain techniques are better suited.
  - Direct parameter

Shifting Frequencies

- Many times it is not possible to acquire all of the data at the same time.
- Small variations in the physical properties of the structure will cause shifts in the modal frequencies.
  - Fuel migrating in wing tanks of aircraft.
  - Thermal expansion due to temperature changes.
- These shifts will cause polyreference to erroneously indicate multiple modes at a single frequency.
- Need to use circle fitting to be able to follow this frequency shifting.